Homework 1: Strings and Languages

Due: Friday, January 21 at 11:59pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF. Please ensure that your work is legible.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it *netid*`-hw1.pdf`, where *netid* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it *netid*`-hw1-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
  - Submit your PDF file(s) in Canvas under Assignments > Homework 1: Strings and Languages.

Problems

1. For each language definition below, describe the language in English giving as concise a description as possible. List two strings that are in L and two that are not (if there are fewer than two strings in L or not in L, then list as many as possible). In all cases, the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.
   (a) $L_1 = \Sigma(\Sigma\Sigma)^*$ **(1 point)**
   (b) $L_2 = \{\mathtt{a}\}\Sigma^*\{\mathtt{a}\} \cup \{\mathtt{a}\}\Sigma^*\{\mathtt{b}\}$ **(1 point)**
   (c) $L_3 = \Sigma^* - \Sigma^*\{\mathtt{bb}, \mathtt{ab}\}$ **(1 point)**
   (d) $L_4 = (\emptyset \cup \{\varepsilon\})^*$ **(1 point)**
   (e) $L_5 = \{w \mid w \in \Sigma^*$ and exactly one prefix of w ends in $\mathtt{a}\}$ **(1 point)**
   (f) $L_6 = \{w \mid w \in \Sigma^*$ and all prefixes of w end in $\mathtt{a}\}$ **(2 points)**
   (g) $L_7 = \{w \mid w \in \Sigma^*$ and $\forall u \in \Sigma^*(|w| \leq |u|)\}$ **(2 points)**
2. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, and let $L = \{w \mid w \in \Sigma^*$ and $\forall u \in \Sigma^*(|w| \geq |u|)\}$. Prove that $L = \emptyset$. **(3 points)**
3. String z is a *substring* of w iff $\exists u, v\ [w = uzv]$. State whether each of the following statements is true or false, and prove your answer.
   (a) For any two strings z and w, if z is a substring of w, then every substring of z is a substring of w. **(3 points)**
   (b) For any two strings z and w, if every substring of z is a substring of w, then z is a substring of w. **(3 points)**
4. For any alphabet Σ, let FINITE be the set of all finite languages over Σ, and let

$$\mathsf{coFINITE} = \{L \mid \overline{L} \in \mathsf{FINITE}\}$$

   (where, for any language L over Σ, $\overline{L}$ is the complement of L, that is, $\Sigma^* - L$). For example, Σ^* is in coFINITE because its complement is $\emptyset$, which is finite. (Please think carefully about this definition, and note that coFINITE isn't the same thing as $\overline{\mathsf{FINITE}}$.)

(a) Are there any languages over Σ in $\mathsf{FINITE} \cap \mathsf{coFINITE}$? Prove your answer. **(3 points)**

(b) Are there any languages over Σ that are *not* in $\mathsf{FINITE} \cup \mathsf{coFINITE}$? Prove your answer. **(3 points)**