Homework 2: Finite Automata

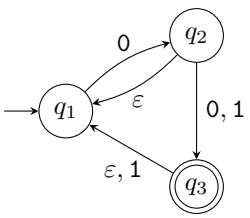
Due: Thursday, January 27 at 11:59pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF. Please ensure that your work is legible.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `netid-hw2.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `netid-hw2-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
  - Submit your PDF file(s) in Canvas under Assignments > Homework 2: Finite Automata.

Problems

1. For each of the languages below, provide a state diagram for a DFA that recognizes it. In all cases, the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.
   (a) $L_1 = \{\mathtt{a}, \mathtt{bb}\}$ **(1 point)**
   (b) $L_2 = \{w \in \Sigma^* \mid \mathtt{ba} \text{ is not a substring of } w\}$ **(1 point)**
   (c) $L_3 = \{w \in \Sigma^* \mid w \text{ does not end in } \mathtt{ba}\}$ **(1 point)**
   (d) $L_4 = \Sigma^* - \{\mathtt{aa}, \mathtt{aba}\}$ **(2 points)**
   (e) $L_5 = \{w \in \Sigma^* \mid \mathtt{baba} \text{ is a substring of } w\}$ **(2 points)**
2. For each of the languages below, provide a state diagram for an NFA that recognizes it. In all cases, the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.
   (a) $L_6 = \{\mathtt{a}, \mathtt{ab}\}$ **(1 point)**
   (b) $L_7 = \{\mathtt{ab}\}^*\{\mathtt{a}\}^*$ **(1 point)**
   (c) $L_8 = \{\mathtt{a}^n(\mathtt{ab})^m \mid n, m > 0 \wedge n \bmod 2 = m \bmod 2\}$ **(2 points)**
   (d) $L_9 = ((L_6)^* L_7)^*$ using L_6 and L_7 above **(3 points)**
3. Convert the following NFA to an equivalent DFA. **(4 points)**



4. Let $\textsc{Interleave}(x, y)$ be an operation defined on strings as

$$\textsc{Interleave}(x, y) = x_1 y_1 x_2 y_2 \cdots x_n y_n$$

where $n = |x| = |y|$, $x = x_1 \cdots x_n$, $y = y_1 \cdots y_n$, and each $x_i, y_i \in \Sigma$. In other words, interleaving two strings of equal length produces a string formed by alternating the symbols in each string. Note that $\textsc{Interleave}(x, y)$ is only defined on strings whose lengths are equal.

Now, let $\textsc{Interleave}(L_1, L_2)$ be an operation defined on languages as

$$\textsc{Interleave}(L_1, L_2) = \{\textsc{Interleave}(x, y) \mid x \in L_1 \wedge y \in L_2 \wedge |x| = |y|\}.$$

Prove that regular languages are closed under $\textsc{Interleave}$. Do this using a proof by construction on DFAs. You may provide either a (precise) description of your construction, or a formal DFA definition. Hint: Simulate both DFAs in parallel, as in the product construction. But you may need to add additional information to achieve the alternating effect. **(6 points)**