Homework 5: Pushdown Automata and Non-context-free Languages

CSE 30151 Spring 2022

Due: Thursday, March 17 at 11:59pm

Instructions

- Use this document to create a PDF file containing your solutions. Do this either by (1) printing this document, writing your solutions on it, and scanning[1] your work into a PDF; or (2) writing your solutions on this PDF digitally. Either way, please ensure that your work is clearly legible.
- If you need extra blank pages, feel free to insert them as needed. The amount of blank space provided beneath a problem does not necessarily indicate the expected length of your solution.
- You have the option to submit your solutions all at once or in parts; late penalties will only be applied to problems that are late. Individual problems cannot be submitted for grading in this way more than once.
- If you plan to submit some parts of your assignment late, before the deadline, upload a single PDF containing the problems you have completed so far. Do not include solutions to problems you want graded later for late credit. After the deadline, if you want to submit additional problems, add them to your original PDF and upload it again.[2]
- Submit your PDF file in Canvas under Assignments > Homework 5: Pushdown Automata and Non-context-free Languages. You may re-submit your work any number of times before the due date.

[1] For tips on scanning your work using your mobile device, see `https://help.gradescope.com/article/0chl25eed3-student-scan-mobile-device`.

[2] For tips on concatenating your old and new PDFs together, see `https://help.gradescope.com/article/tp9kl4yx4q-student-troubleshooting-submissions`.

1. For each of the following languages, provide a state diagram for a PDA that recognizes it. The notation $c_a(w)$ indicates the number of occurrences of symbol a in string w.

   (a) $L_1 = \{\mathtt{a}^n\mathtt{b}^m\mathtt{a}^n \mid n, m \geq 0 \text{ and } m \text{ is even}\}$ **(2 points)**

(b) $L_2 = \{x^R \texttt{\#} y \mid x, y \in \{\texttt{0}, \texttt{1}\}^*$ and x is a substring of $y\}$ **(2 points)**

(c) $L_3 = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid c_{\mathtt{a}}(w) = c_{\mathtt{b}}(w)\}$ **(2 points)**

(d) $L_4 = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid c_{\mathtt{a}}(w) = 2 \cdot c_{\mathtt{b}}(w)\}$ **(2 points)**

2. Convert each of the following CFGs to an equivalent PDA.

   (a) $S \to \mathtt{a}S\mathtt{a} \mid T \mid \varepsilon$
   $T \to \mathtt{b}T \mid \mathtt{c}T \mid \varepsilon$

   (2 points)

(b) $S \to \mathtt{a}AC\mathtt{a}$
$A \to B \mid \mathtt{a}$
$B \to C \mid \mathtt{c}$
$C \to \mathtt{c}C \mid \varepsilon$

(2 points)

3. Prove that each of the following languages is not context-free.

   (a) $L_5 = \{(\mathtt{ab})^n \mathtt{a}^n \mathtt{b}^n \mid n > 0\}$ **(3 points)**

(b) $L_6 = \{wxw \mid |w| = 2 \cdot |x| \text{ and } w \in \{\mathtt{a}, \mathtt{b}\}^* \text{ and } x \in \{\mathtt{c}\}^*\}$ **(3 points)**

(c) $L_7 = \{\mathtt{a}^n\mathtt{b}^m\mathtt{c}^k \mid n, m, k \geq 0 \text{ and } m \leq \min(n, k)\}$ **(3 points)**

(d) $L_8 = \{ww^R w \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$ **(3 points)**